Name _____

Pre-Calculus Second Semester Exam Review

You may turn in this exam review for 3% bonus on your exam if all work is shown (correctly) and answers are correct. Please show work NEATLY and box in or circle the answers. If you do not have correct work and answers for any problem, you will not get the extra credit!

Chapter 8 and 7.5

Given that ABCD is a parallelogram with AB = 6, BC = 10 and m$\angle$A = 60°, complete the following.

1) $\overrightarrow{AB} + \overrightarrow{BC} =$

2) $\left|\overrightarrow{AB} + \overrightarrow{BC}\right| =$

3) $\left|\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}\right| =$

4) $\overrightarrow{AB} + \overrightarrow{AD} =$

5) $\left|\overrightarrow{BD} + \overrightarrow{DC}\right| =$

For problems 6 – 15, let $\mathbf{u} = \langle -4, 3 \rangle$ and $\mathbf{v} = \langle -5, 12 \rangle$. Calculate each of the following.

6) $\mathbf{u} + \mathbf{v} =$

7) $2\mathbf{u} + \mathbf{v} =$

8) $\mathbf{u} - 3\mathbf{v} =$

9) $|\mathbf{u} + 2\mathbf{v}| =$

10) $\mathbf{u} \bullet \mathbf{v}$

11) $|\mathbf{u}|$

12) $|\mathbf{v}|$

13) Determine θ, the measure of the angle between **u** and **v**, to the nearest tenth.

14) Determine the value of *a* such that the vector $\langle a, 6 \rangle$ is parallel to **u**.

15) Determine the value of *a* such that the vector $\langle 6, a \rangle$ is perpendicular to **u**.

For problems 16-21, let A = (4,-6), B = (0,-1) and C = (3,2).

16) Determine the midpoint of $\overline{AB}$.

17) $\overrightarrow{AB}$ =

18) $\left|\overrightarrow{AB}\right|$ =

19) Are any two of the vectors formed by these points perpendicular? Show all 3 vectors and work!

20) Name a vector perpendicular to $\overrightarrow{AB}$.

21) Name a vector parallel to $\overrightarrow{AB}$.

22) Write the pair of parametric equations $x = 3\sin\theta$ and $y = 3\cos\theta$ in rectangular form.

23) Write the pair of parametric equations $x = \cos\theta$ and $y = 4\sin\theta$ in rectangular form.

24) F_1 is a force 3N pulling an object north and F_2 is a force of 5N pulling an object 60° east of north. Determine the direction (measured clockwise from north) and magnitude of the resultant force.

25) A plane is on a course of S80°W at a speed of 320 mph. What are the north-south and east-west components of the plane's velocity vector?

26) Find the vertical component of the vector **v** with a magnitude of 58 feet and a direction angle of 30° from the horizontal.

Chapter 10

For each of the following sequences, determine the value of a_5 and a_6 and write an explicit rule for a_n. State whether the sequence is arithmetic, geometric or neither.

27) 23, -23, 23, -23, …

28) 101, 91, 81, 71, …

29) 4, -16, 64, -256, …

30) 120, 60, 30, 15, …

31) 3, 6, 11, 18, …

32) 3, 7, 11, 15, …

Determine the first four terms of the following sequences. State whether the sequence is arithmetic, geometric or neither.

33) $a_1 = 12$ and $a_n = 3a_{n-1} - 4$

34) $a_n = n + \frac{1}{n}$

35) $a_n = 3n + 2$

36) $a_n = 3^n$

37) $a_1 = 5, a_2 = 8, a_n = a_{n-1} + a_{n-2}$

Evaluate the following sums. You **must** show work!

38) $\sum_{k=1}^{5} 3k - 2$

39) $\sum_{k=1}^{4} 2 \cdot 3^{k-1}$

40) $\sum_{k=1}^{\infty} 4\left(-\frac{1}{3}\right)^{k-1}$

Write the following using sigma notation.

41) $1 + 8 + 27 + 64 + 125$

42) $5 + 10 + 15 + 20$

43) $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+...$

Determine the specified sum/term for each of the following series.

44) Determine the 10^{th} partial sum for the arithmetic series with $a_1 = 5 \text{ and } a_{10} = 41$.

45) Determine the 4^{th} partial sum for the geometric series with $a_1 = 1 \text{ and } a_2 = 4$.

46) Determine the 9^{th} partial sum for the geometric series with $a_1 = 6$ and $r = -2$.

47) S_{50} for the series with $a_1 = 4$ and $a_n = a_{n-1} + 2$

48) Determine a_{60} of an arithmetic sequence with $a_4 = 7 \text{ and } a_7 = 13$

49) Determine a_6 of a geometric sequence with $a_1 = 16$ and $a_4 = 54$.

50) An arithmetic series has first term 78 and common difference 3.2. Determine the thirtieth term and thirtieth partial sum.

51) An arithmetic series has $a_1 = 56$ and $a_3 = 66$. Determine a_2 and S_{100}.

52) A geometric series has $a_1 = 100$ and $a_2 = 90$. Determine the value of $a_3, a_4,$ and a_5. Evaluate S_{20}. Determine the limit to which the partial sums converge.

53) An arithmetic series has first term 13 and tenth term 31. What is the twentieth term?

54) What are the next two terms of the sequence $\frac{1}{3}, \frac{2}{9}, \frac{3}{27}, \frac{4}{81}, \ldots$? Is this sequence arithmetic, geometric or neither?

55) Write using sigma notation and determine the sum: $\frac{1}{5} + \frac{1}{25} + \frac{1}{125} + \frac{1}{625} + \ldots$

56) Evaluate S_{20} for the geometric series 100 + 88 + 77.44 +… Also, determine the limit of this series.

57) Explain why the geometric series 10 + 20 + 40 + 80 + … does not converge.

58) What is the fifth term in the expansion $(2x-3y)^4$?

59) The expression $x^5+10x^4y+40x^3y^2+80x^2y^3+80xy^4+32y^5$ is the expansion a binomial $(a+b)^n$. Determine a, b, and n.

Chapter 6

Evaluate. You **must** show work!

60) $\begin{vmatrix} 5 & -2 \\ 4 & 6 \end{vmatrix}$

61) $\begin{vmatrix} 5 & -1 & 3 \\ 0 & 1 & 2 \\ -6 & 1 & 4 \end{vmatrix}$

62) $\begin{vmatrix} 2 & 0 & 5 \\ -3 & 0 & -4 \\ 6 & 3 & 1 \end{vmatrix}$

Perform the following operations, if possible. Use the matrices given below.

$$A=\begin{bmatrix}-1 & a\\ 0 & 4\end{bmatrix} \qquad B=\begin{bmatrix}6\\ -1\\ b\\ 2\end{bmatrix} \qquad C=\begin{bmatrix}7 & -2\\ 0 & c\\ -3 & d\end{bmatrix} \qquad D=\begin{bmatrix}-3 & 1\\ -2 & 4\end{bmatrix}$$

63) A + B

64) 2A – D

65) D – C

66) A^{-1}

67) CD

68) AD

69) BC

Solve for the variables. You **must** show work!

70) $3\begin{bmatrix}x+4\\ y-2\end{bmatrix}+\begin{bmatrix}5\\ -4\end{bmatrix}=\begin{bmatrix}8\\ 2\end{bmatrix}$

71) $\begin{bmatrix}1 & 1 & 3\\ 2 & -1 & 4\\ 3 & 0 & -2\end{bmatrix}\begin{bmatrix}1\\ -2\\ x\end{bmatrix}=\begin{bmatrix}5\\ y\\ -1\end{bmatrix}$

72) $\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix}\mathbf{X}=\begin{bmatrix}-3 & 4\\ 1 & 2\end{bmatrix}$

73) $\begin{bmatrix}9 & 7\\ 5 & 4\end{bmatrix}\mathbf{X}=\begin{bmatrix}1 & 0 & -2\\ 4 & 1 & 3\end{bmatrix}$

74) $\begin{bmatrix}6 & 2\\ 4 & -2\end{bmatrix}\mathbf{X}-\begin{bmatrix}2 & -2\\ 3 & 1\end{bmatrix}=\begin{bmatrix}6 & 4\\ 8 & -2\end{bmatrix}$

75) What is the augmented matrix for the system $\begin{matrix} 2x - y = 24 \\ x + 6y = 52 \end{matrix}$?

Solve each system using matrices. **Show** the matrix (or matrix equation) you use to solve. Write your answer as an ordered pair or ordered triple.

76) $\begin{matrix} 2x - 5y = 4 \\ x - 3y = 1 \end{matrix}$

77) $\begin{matrix} 3x - 7y = -16 \\ -2x + 3y = 9 \end{matrix}$

78) $\begin{matrix} x + 3y - 2z = -11 \\ -3x + 10y - 7z = -13 \\ -2x - 6y + 5z = 27 \end{matrix}$

79) $\begin{matrix} x - 3y + 2z = -6 \\ -3x - 10z = 6 \\ 5x + 6y - 2z = 14 \end{matrix}$

Determine the inverse of the following matrices, if it exists. You **must** show work!

80) $\begin{bmatrix} 10 & -2 \\ -6 & 1 \end{bmatrix}$

81) $\begin{bmatrix} 4 & -9 \\ 0 & 7 \end{bmatrix}$

82) $\begin{bmatrix} 6 & -8 \\ -3 & 4 \end{bmatrix}$

83) $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

Chapter 12 & Section 2-5

Evaluate the limit or state that it does not exist.

84) $\lim_{x\to-\infty}\dfrac{3x^3-8x^2+6}{9-15x^5}$

85) $\lim_{x\to6}\dfrac{x^2-36}{x^2+4x-60}$

86) $\lim_{x\to3}\dfrac{x-3}{x^2-9}$

87) $\lim_{x\to2^+}\dfrac{3x+1}{2-x}$

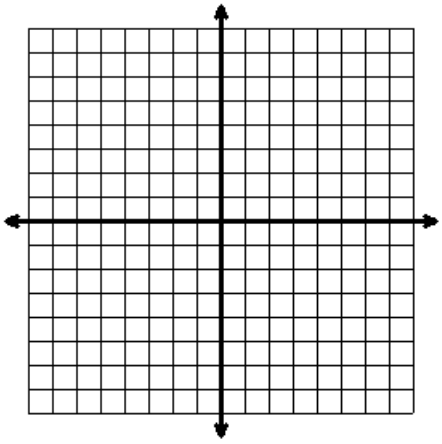
88) $\lim_{x\to-5}\dfrac{1}{x+5}$

89) $\lim_{n\to\infty}\dfrac{3n^2-n-2}{2n^2+5n}$

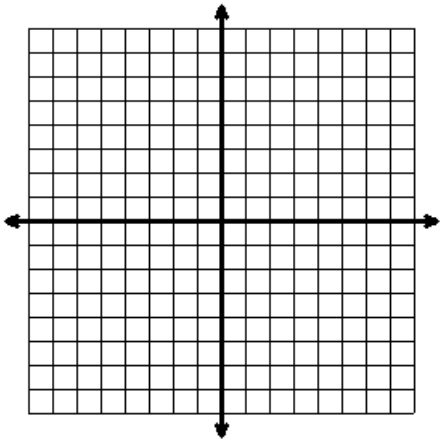
90) $\lim_{x\to4}\dfrac{\sqrt{x}-2}{x-4}$

Sketch the graph. Name the intercepts, asymptotes and list any discontinuities.

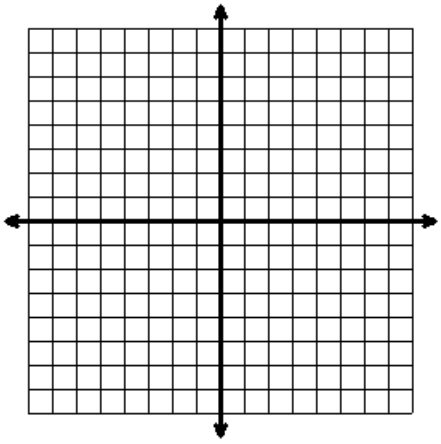
91) $y=\dfrac{x^2+x}{3x-6}$



92) $y=\dfrac{8}{x^2-9}$



93) $y=\dfrac{x^2+x-6}{x^2-4}$



Chapter 9

94) Determine two other polar representations of the following points. Show work or a sketch.

a) $\left(-4, 100^\circ\right)$ b) $\left(2, \pi\right)$ c) $\left(-3, 225^\circ\right)$

95) Change to polar coordinates. Show work or a sketch.

a) (0,-3) b) (-4,-4)

96) Change to rectangular coordinates. Show work or a sketch.

a) $\left(4, 240^\circ\right)$ b) $\left(-3, \frac{\pi}{2}\right)$ c) $\left(4, -\frac{\pi}{4}\right)$

97) Convert the polar equation to rectangular form.

a) $r = 10\sec\theta$ b) $r = 3\cos\theta$

98) Convert the rectangular equation to polar form.

a) x = 10 b) 3x + 2y = 7 c) $x^2 + y^2 = 16$

99) Express the complex number in polar form. Show work or a sketch.

a) $-2 - 2i$ b) $3i$ c) $1 - i\sqrt{3}$

100) Express the complex number in rectangular form. Show work or a sketch.

a) $4\text{cis}240^\circ$ b) $6\text{cis}\frac{5\pi}{3}$

101) Evaluate the product in polar form and convert your answer to rectangular form:
$\left(4\text{cis}60^\circ\right)\left(3\text{cis}30^\circ\right)$

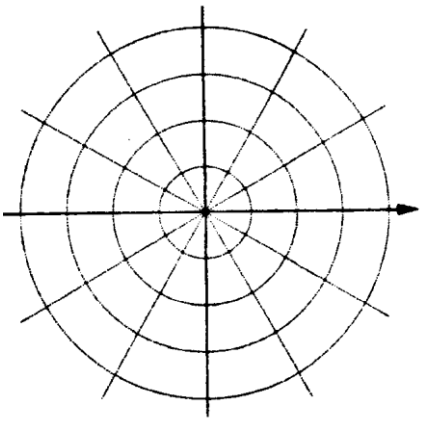
102) Plot the points on a polar graph.

a) $A\left(2,60^\circ\right)$

b) $B\left(-4,300^\circ\right)$

c) $C\left(4,-\pi\right)$

d) $D\left(-1,\frac{5\pi}{3}\right)$



103) Simplify $\left(3+3i\right)^4$. You **must** show work!

104) Given $z_1 = 4cis30^\circ$ and $z_2 = \frac{1}{8}cis120^\circ$. Determine each of the following in polar form.

a) $z_1 z_2$ b) $\left(z_1\right)^7$ c) $\left(z_2\right)^2$

105) Determine all solutions to the equation $x^4 + 81 = 0$. Leave answers in polar form.

106) Determine the fifth roots of $-i$. Leave answers in polar form.

107) Determine the cube roots of $4cis60°$ in polar form.