2) Prove $\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$

This is saying $t_n = n(n+1)$, Prove $S_n = \frac{n(n+1)(n+2)}{3}$

When $n=1$, $t_1 = S_1 = 1(1+1) = 1\cdot 2 = 2$ so this is true.

$$S_1 = \frac{1(1+1)(1+2)}{3} = \frac{1\cdot 2\cdot 3}{3} = 2$$

Assume this is true when $n=k$.
In other words, $S_k = \frac{k(k+1)(k+2)}{3}$.
I want to show this is true
when $n=k+1$:

$$S_{k+1} = S_k + t_{k+1}$$

Common denominator

Factor out $(k+1)(k+2)$

$$= \frac{k(k+1)(k+2)}{3} + (k+1)(k+1+1)$$

$$= \frac{k(k+1)(k+2)}{3} + \frac{3(k+1)(k+2)}{3}$$

$$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3}$$

$$= \frac{(k+1)(k+2)[k+3]}{3}$$

$$= \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$$

$\therefore S_n = \frac{n(n+1)(n+2)}{3}$ for all n.