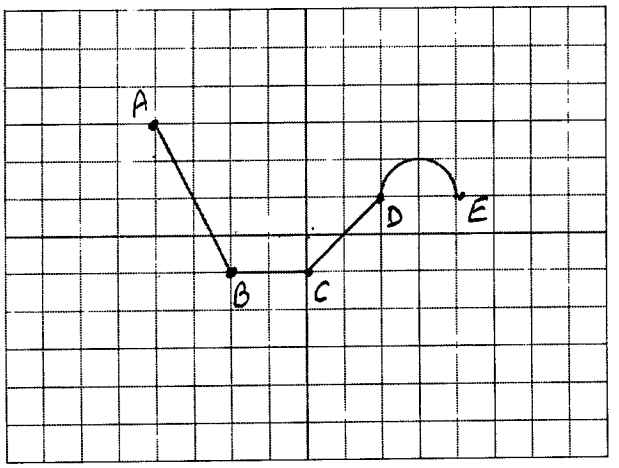
Name Key Hr___

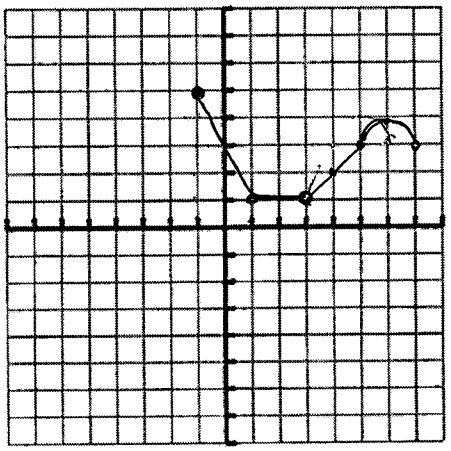
Honors PreCalc – Chapter 1 Homework Packet

1) The graph of $y = f(x)$ is shown. Sketch the graph of each transformation.

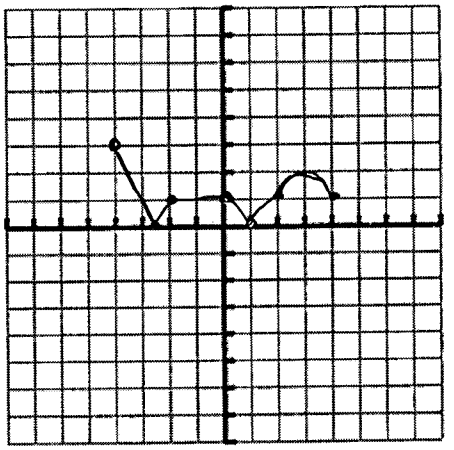
A(-4,3)
B(-2,-1)
C(0,-1)
D(2,1)
E(4,1)



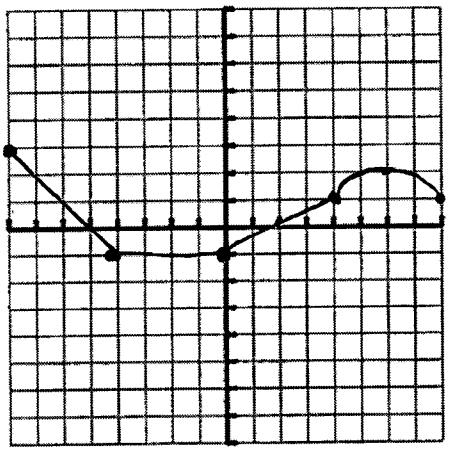
a) $y = f(x-3)+2$ right 3 up 2



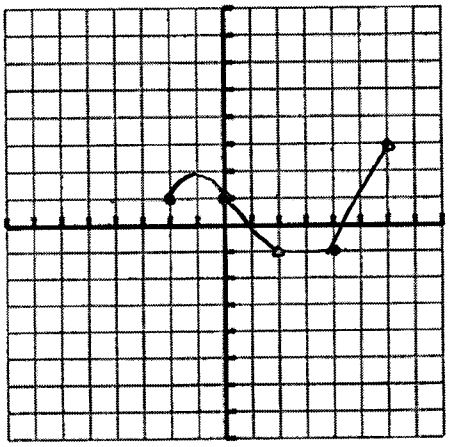
b) $y = |f(x)|$



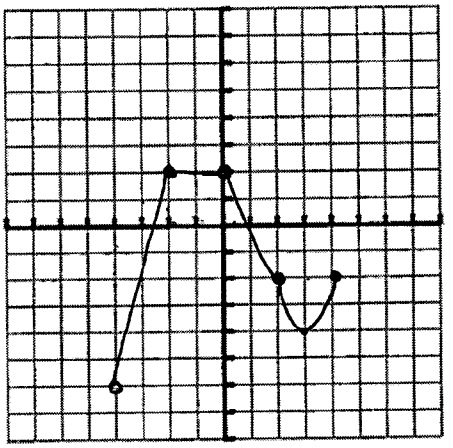
c) $y = f(\frac{1}{2}x)$



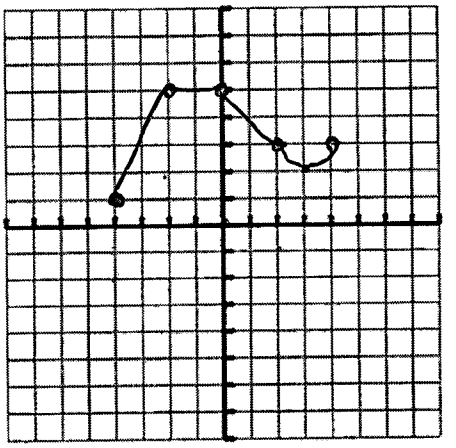
d) $y = f(-x+2)$ $= f(-(x-2))$



e) $y = -2f(x)$



f) $y = -f(x)+4$



2) Suppose $f(x)=\lfloor x \rfloor$. Evaluate each of the following.

a) Domain of $f(x)$. $\mathbb{R}$

b) Range of $f(x)$. $\mathbb{Z}$

c) Zero(s) of $f(x)$. $[0,1)$

d) $f(-4.6) = -5$

e) $f\left(\frac{-82}{3}\right) = -28$

f) $f(\pi) = 3$

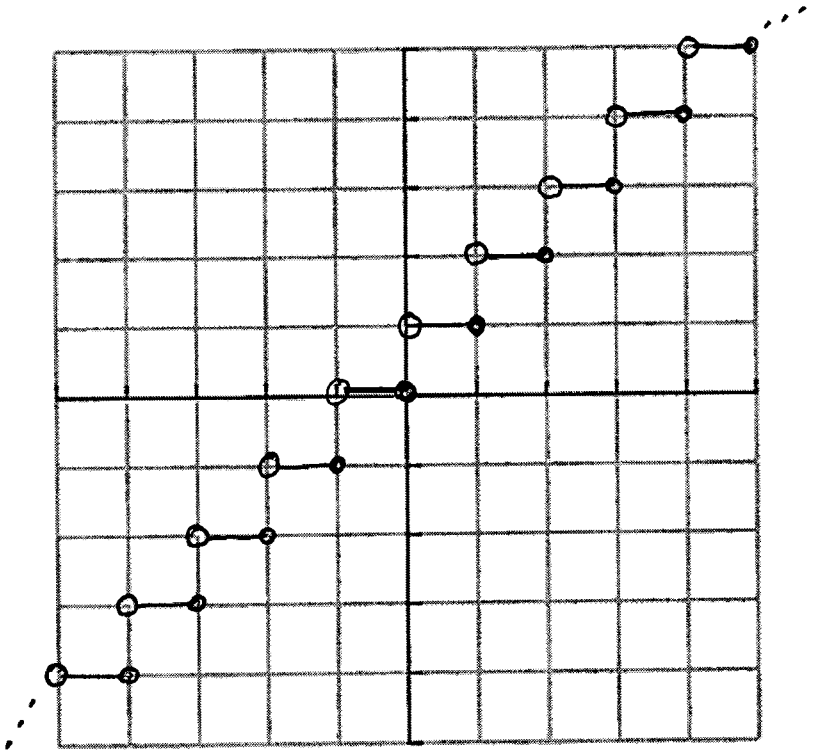
3) The least integer function can be defined in a similar way as the greatest integer function. The notation used is $f(x)=\lceil x \rceil$. It is also referred to as the ceiling function.

a) In words, describe how to find $f(x)$ for a given x value.

f(x) is the smallest integer greater than or equal to x.

b) Fill out the table and then graph the least integer function.

x	$f(x)$
5.8	6
17.1	18
6	6
¼	1
0	0
-8.25	-8
-17	-17
π	4
e	3
$\sqrt{2}$	2

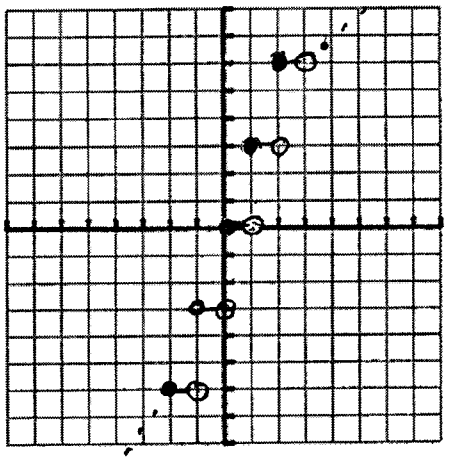


4) Graph $f(x)=3\lfloor x \rfloor$. Identify the domain, range and zeros of the function.

$D=\{x \mid x \in \mathbb{R}\}$

$R=\{y \mid y \text{ is a multiple of } 3\}$

zeros: $[0,1)$



5) A taxicab charges a flat fee of \$1.75 plus \$0.30 for each half-mile or fraction of a half-mile.

a) Fill in the chart.

Time	0.7 miles	1.23 miles	3 miles	10.9 miles
Cost	\$2.35	\$2.65	\$3.55	\$8.35

b) Write an equation for the function.

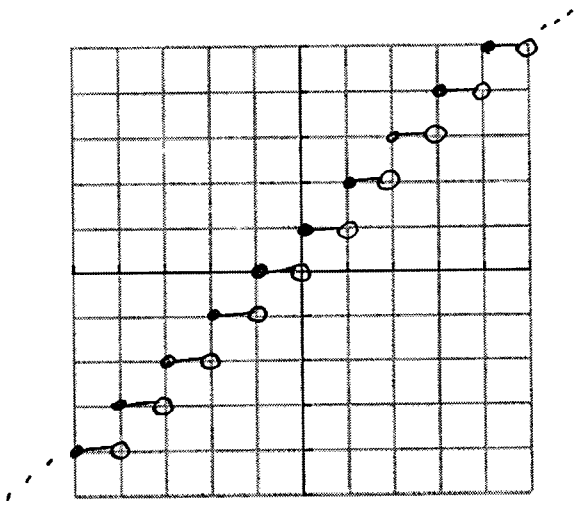
$f(x)=1.75+0.3\lceil 2x \rceil$

6) OfficeMax.com sells pencils to individuals and to organizations such as schools or businesses. For up to ten boxes of pencils, they cost \$1.70 per box and \$3 shipping and handling. If a school wants to order more than 10 and less than 50 boxes of pencils, they get a 10% discount on the pencils and must pay \$10 shipping and handling. If an organization buys 50 boxes or more, they get free shipping and get a 12% discount on the pencils. Write a function to represent this scenario.

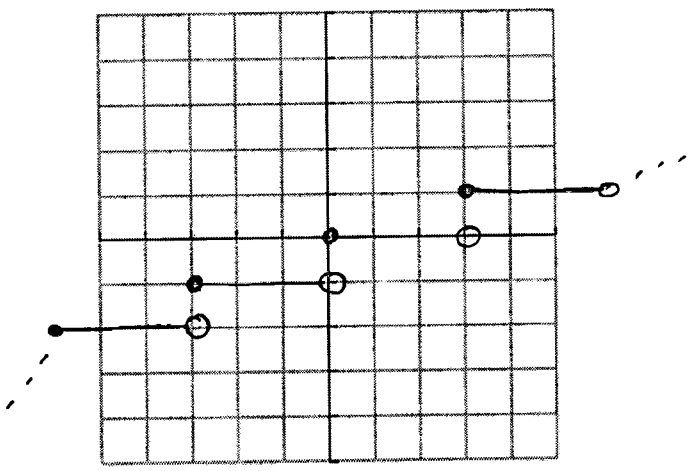
$$f(x)=\begin{cases} 1.7x+3 & 0<x\leq 10 \\ 1.53x+10 & 10<x<50 \\ 1.496x & x\geq 50 \end{cases}$$

7) Sketch the graph.

a) $g(t)=[\![t+1]\!]$



b) $y=\left\lfloor \frac{1}{3}x \right\rfloor$

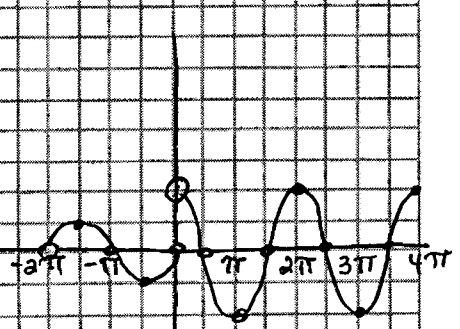


c) $f(x)=\begin{cases}(x+3)^2 & x<-3\\ |x| & -3<x\le 2\\ -x+1 & x>2\end{cases}$

d) $g(x)=\begin{cases}\sin x & -2\pi<x\le 0\\ 2\cos x & 0<x\le 4\pi\end{cases}$

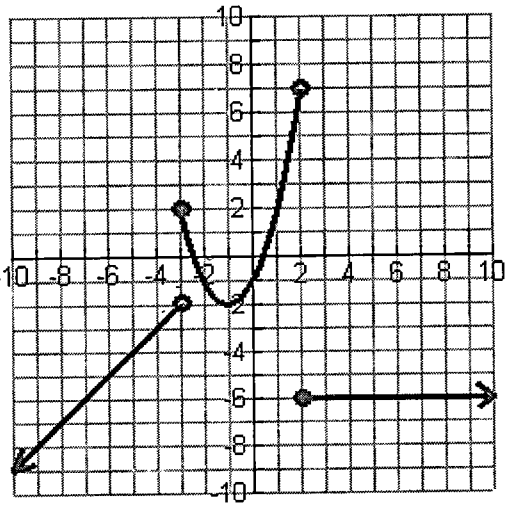
c)

d)



8) Write the equation of the graph shown.

$$f(x)=\begin{cases}x+1 & x<-3\\ (x+1)^2-2 & -3\le x<2\\ -6 & x\ge 2\end{cases}$$



9) A car traveling x miles per hour stops quickly. The distance (in feet) a car travels during the driver's reaction time is given by $R(x) = \frac{3}{4}x$. The distance (in feet) traveled while braking is given by $B(x) = \frac{1}{15}x^2$.

a) Determine the stopping-distance function T.

$T(x) = \frac{3}{4}x + \frac{1}{15}x^2$

b) Which function contributes most to the magnitude of the sum at higher speeds? Explain.

$B(x)$ since it has a squared term. The value of $B(x)$ will be greater than $R(x)$ for most values of x.

10) Suppose $f(x) = 3x - 5$, $g(x) = -x^2 - x - 1$ and $h(x) = \sqrt{x}$.

a) Fill out the following chart.

function	equation	domain
$(f+h)(x)$	$(f+h)(x) = \sqrt{x} + 3x - 5$	$[0, \infty)$
$(f-g)(x)$	$(f-g)(x) = x^2 + 4x - 4$	$\mathbb{R}$
$(fh)(x)$	$(fh)(x) = 3x^{\frac{3}{2}} - 5x^{\frac{1}{2}}$	$[0, \infty)$
$(h/f)(x)$	$\left(\frac{h}{f}\right)(x) = \frac{\sqrt{x}}{3x-5}$	$[0, \frac{5}{3}) \cup (\frac{5}{3}, \infty)$
$(h \circ f)(x)$	$(h(f(x)) = \sqrt{3x-5}$	$[\frac{5}{3}, \infty)$
$(f \circ h)(x)$	$f(h(x)) = 3\sqrt{x} - 5$	$[0, \infty)$

b) Evaluate each of the following.

i. $\left(\frac{g}{h}\right)(16) = \frac{-273}{4}$

ii. $\left(\frac{f}{h}\right)(-4) =$ undefined

iii. $(h-f)(1) = 3$

iv. $f(f(3)) = 7$

11) Suppose the weekly operating costs of a company to produce x units of a product is approximated by $f(x)$. The weekly sales of selling x units is approximated by $g(x)$. What would $(g-f)(x)$ mean in this context?

The would represent the weekly profit of the company.

12) Three siblings are three different ages. The oldest is twice the age of the middle sibling and the middle sibling is six years older than one half the age of the youngest. Write a composite function that gives the oldest sibling's age in terms of the youngest.

$D(m) = 2m$

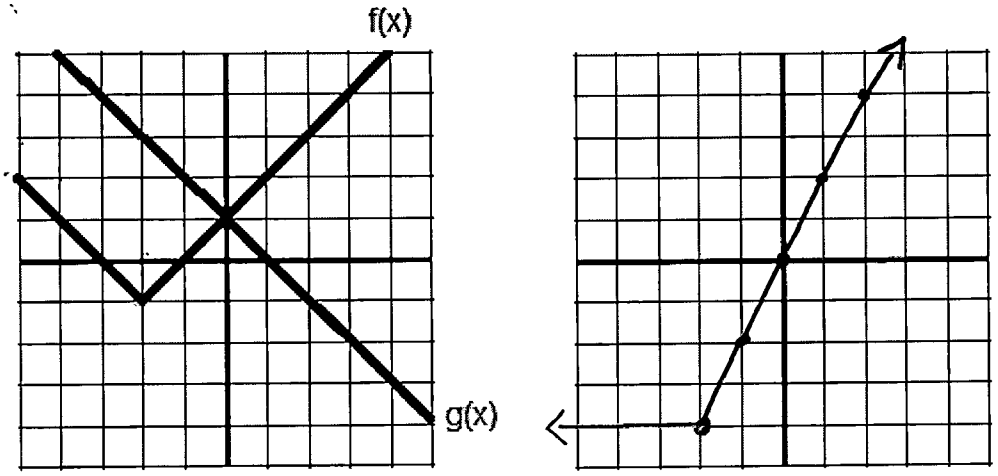
$m(y) = \frac{1}{2}y + 6$

$D(m(y)) = 2\left(\frac{1}{2}y + 6\right)$

$= y + 12$

$\boxed{D(y) = y + 12}$

13) In the graph below, $f(x)$ is an absolute value function and $g(x)$ is a linear function. On the blank graph, sketch the graph of $(f - g)(x)$.



14) Laura sends an average of 27 text messages per month to each of f friends. Her cell phone provider charges her a flat rate of \$3.50 per month and \$0.04 per text message. The function $t(f)$ gives the total number of text messages Laura sends each month to f friends, and $g(t)$ gives the amount Laura is charged by her cell phone provider for t text messages.

a) Write an equation for $t(f)$ and $g(t)$. $t(f) = 27f$

$g(t) = 3.5 + 0.04t$

b) Write an equation for $g(t(f))$. What does it mean in the context of this problem?

$g(t(f)) = 3.5 + 0.04(27f)$

$= 1.08f + 3.5$

This is the total cost of her cell phone each month.

15) Suppose $f(x) = \sqrt{x-7}$ and $g(x) = x^2 + 19$. What is $g(f(x))$?

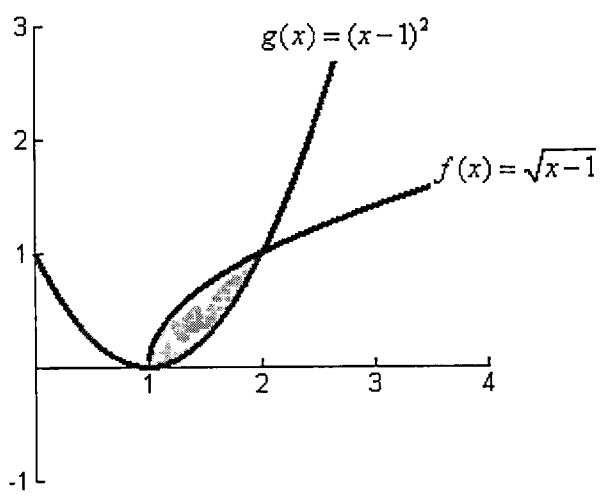
$g(f(x)) = x + 12, \quad x \geq 7$

16) Suppose $f(x) = -2x - 9$. What is $f^{-1}(x)$? $f^{-1}(x) = \frac{x+9}{-2}$

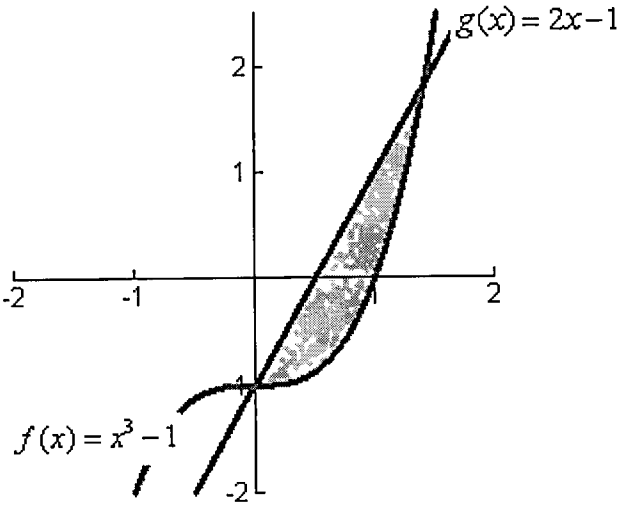
17) Suppose $f(x) = x^2 + 6$. What is $f^{-1}(x)$? does not exist (w/o a domain restriction on $f(x)$)

18) Determine the *y*-value that produces the greatest horizontal distance between the two functions in the shaded region. Then, find the distance. SKIP

a)



b)



19) Determine the inverse of each function.

a) $f(x)=\frac{2}{7}x-9$

$f^{-1}(x)=\frac{7}{2}x+\frac{63}{2}$

b) $f(x)=x^5-1$

$f^{-1}(x)=\sqrt[5]{x+1}$

c) $f(x)=10^x$

$f^{-1}(x)=\log x$

d) $f(x)=\frac{4}{x-3}$

$f^{-1}(x)=\frac{4}{x}+3$

e) $y=\ln x$

$f^{-1}(x)=e^x$

f) $y=\sqrt[3]{2x}$

$f^{-1}(x)=\frac{1}{2}x^3$

20) If $f^{-1}(x)$ and $f(x)$ are inverse functions, then $f(f^{-1}(x))=x$ and $f^{-1}(f(x))=x$. Verify that the functions $g(x)=\frac{x+3}{x-2}$ and $h(x)=\frac{2x+3}{x-1}$ are inverse functions.

$g(h(x))=\dfrac{\frac{2x+3}{x-1}+3}{\frac{2x+3}{x-1}-2}$

$=\dfrac{\frac{2x+3+3x-3}{x-1}}{\frac{2x+3-2x+2}{x-1}}$

$=5x-x$

$h(g(x))=\dfrac{2\left(\frac{x+3}{x-2}\right)+3}{\frac{x+3}{x-2}-1}$

$=\dfrac{\frac{2x+6+3x+6}{x-2}}{\frac{x+3-x+2}{x-2}}$

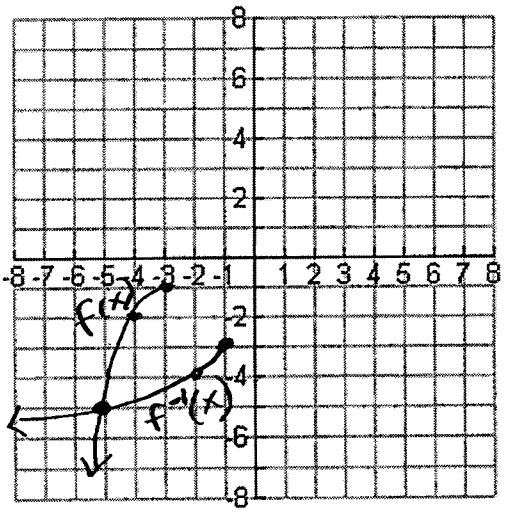
$=\frac{5x}{5}$

$=x$

$\therefore g(x)$ and $h(x)$ are inverse functions

21) Let $f(x) = -(x+3)^2 - 1$ for $x \leq -3$. Find $f^{-1}(x)$ and graph both functions. How do the graphs of inverse functions relate to each other?

They are a reflection over the line $y = x$.



22) Decide if each family of functions is *always*, *sometimes*, or *never* one-to-one.

a) Linear: S

b) Quadratic: N

c) Exponential: A

d) Absolute Value: N

e) Square Root: A

23) Let $f(x) = -2x + 3$ and $g(x) = \frac{3}{x} + 1$. What is $(f^{-1} \circ g^{-1})(8)$? Leave as an exact answer. Show work!

$\frac{9}{7}$

24) Use the table of values to find the following values or say that it cannot be determined.

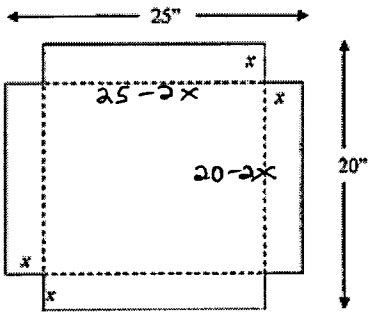
a) $f^{-1}(0) = 4$

b) $f(f(0)) = -3$

c) $f^{-1}(f^{-1}(6)) = 4$

x	$f(x)$
-2	9
0	6
2	3
4	0
6	-3
8	-6

25) An open box is to be made from a rectangular piece of material (25 inches by 20 inches) by cutting equal squares from the corners and turning up the sides.



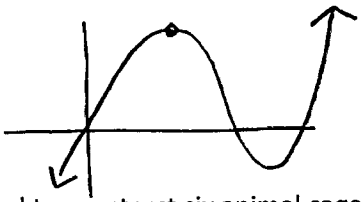
a) What function represents the volume of the box as a function of the side length of the square cutouts?

$V(x) = x(25-2x)(20-2x)$

b) What is the relevant domain of the volume function?

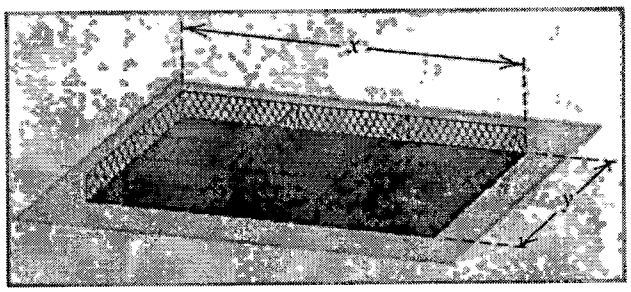
$D: (0, 10)$

c) Sketch a graph of the volume function? What is the maximum volume that can be produced? How did you arrive at that answer?



$V(x) = 820.5$

26) One thousand feet of fencing is used to construct six animal cages, as shown below. Show work!



a) Express the width y as a function of the length x.

$y = 250 - \frac{3}{4}x$

b) Express the total enclosed area A of the cages as a function of x.

$A(x) = x(250 - \frac{3}{4}x)$

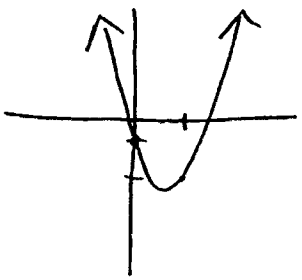
c) Find the length and width that maximizes the enclosed area.

length $= x = 166\frac{2}{3}$ ft

width $= y = 125$ ft.

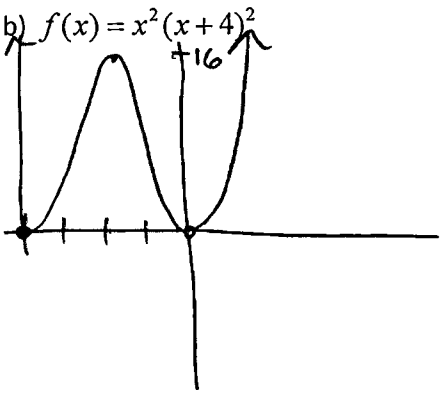
27) Sketch a graph of each function. Identify any extrema. Classify as a relative or absolute max or min.

a) $y = 2x^2 - 3x - 1$



abs. min. (0.75, -2.125)

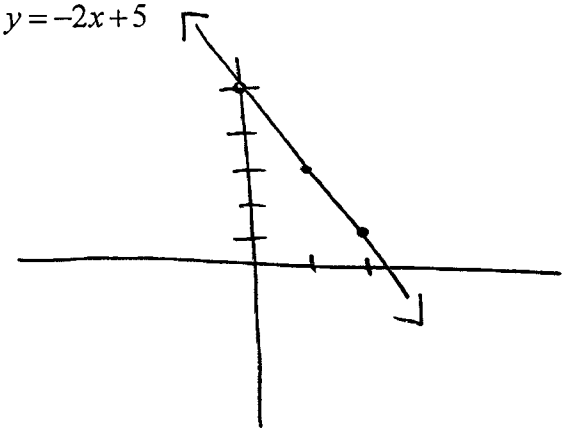
b) $f(x) = x^2(x+4)^2$



abs. mins.: (-4, 0) and (0, 0)

rel. max.: (-2, 16)

c) $y = -2x + 5$



no max. or min. values

28) The equation $P(x) = .002x^4 - .062x^3 + .379x^2 - .194x + 15.256$ estimates the profit of a company (in millions of dollars) during the years 2000 to 2013 where x = the number of years past 2000. During which year did the highest sales occur? What were the sales?

During the year 2004, the highest profit is $\approx$ \$17.26 million.

29) How many extrema does a quadratic function have? What about a cubic function?

A quadratic has one and a cubic has at most two.